How can you read this? There's no pictures!

Druhan Shah

Prerequisites

What is this, 3B1B?

Nifty definitions

Send Cat pics

It's the Ice Cream Man!

Breaking all Limits

Better than the New Acad Block

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Diagrams and their limits in Category Theory

Druhan Shah

September 8, 2025

Outline

Prerequisites

What is this, 3B1B?

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Better than the New Acad Block

1 Prerequisites

2 What is this, 3B1B?

3 Nifty definitions

4 Send Cat pics

5 It's the Ice Cream Man!

6 Breaking all Limits

7 Better than the New Acad Block

Real Analysis PTSD

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Breaking all Limits

Better than the New Acad Block

- Sequences
- Countable sets
- Indexing
- Absolutely no smartness

Shoulders of Giants

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Breaking all Limits

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- Category Theory in Context
- Youtube user Sheafification of g (Warning: Goes balls deep from the get go)
- Math3ma's blog (Absolutely beautiful, has some nice insights into other fields)

A property, and Superman

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- What's the most even number?
- What's the smallest natural number?
- Why are we able to define superlatives for such properties?
- We now have an idea for something that "uniquely" describes a specific structure.

Ducks in a line

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- What is a sequence $\langle x_i \rangle_{i \in \mathbb{N}}$ where $x \in S$?
- if not an injective map $f : \mathbb{N} \to S$?
- Shouldn't something like $\langle x_i \rangle_{i \in A}$ for an arbitrary A make sense?

Shape of U

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For any indexed structure like a set or field or something, it would need a "shape". Countable sequences are said to have the same "shape as $\mathbb{N}$". So, for an indexing $f : J \to A$, we say that the image of that indexing (a substructure of A) has shape J.

List all the Congruence Rules (4 marks)

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Isomorphisms are special cases of these, where we say that the indexed object is "congruent" in a sense to the shape object. This is also how we determine countability of sets, and observe rank-like behaviour in free groups and vector spaces.

Insert abstract nonsense joke here

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Basic Category Theory

- Categories $\mathcal{A} = (\mathrm{Ob}(\mathcal{A}), \mathrm{Hom}(\mathcal{A}))$
- Functors $F : \mathcal{A} \to \mathcal{B}$
- Natural Transformations
- Isomorphism
- Equivalence
- Adjoints (okay, maybe not this)

Now that's a lotta constraints (Flex tape can fix that)

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- Notice how equations in a certain domain strictly use operations that make sense for that domain (duh)
- Group theory equations are a sheaf-load of multiplication
- Indeed, all category theory equations are a metric func-ton of compositions

Something something picture something something thousand words

How can you read this? There's no pictures!

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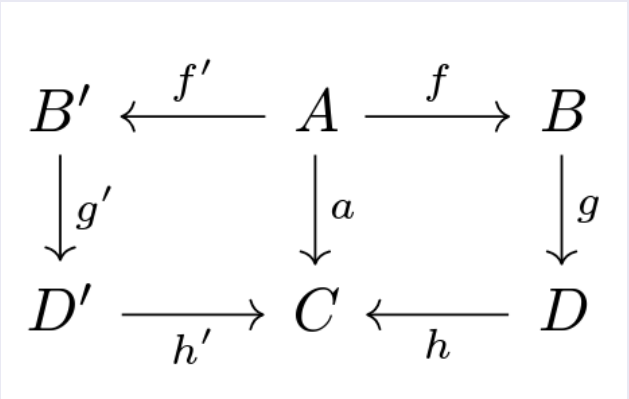
Look at this graph!



The proverbial thousand words:

- $h \circ g \circ f = h' \circ g' \circ f'$
- Every other possible pair of composed paths

Absolute state of this

How can you read this? There's no pictures!

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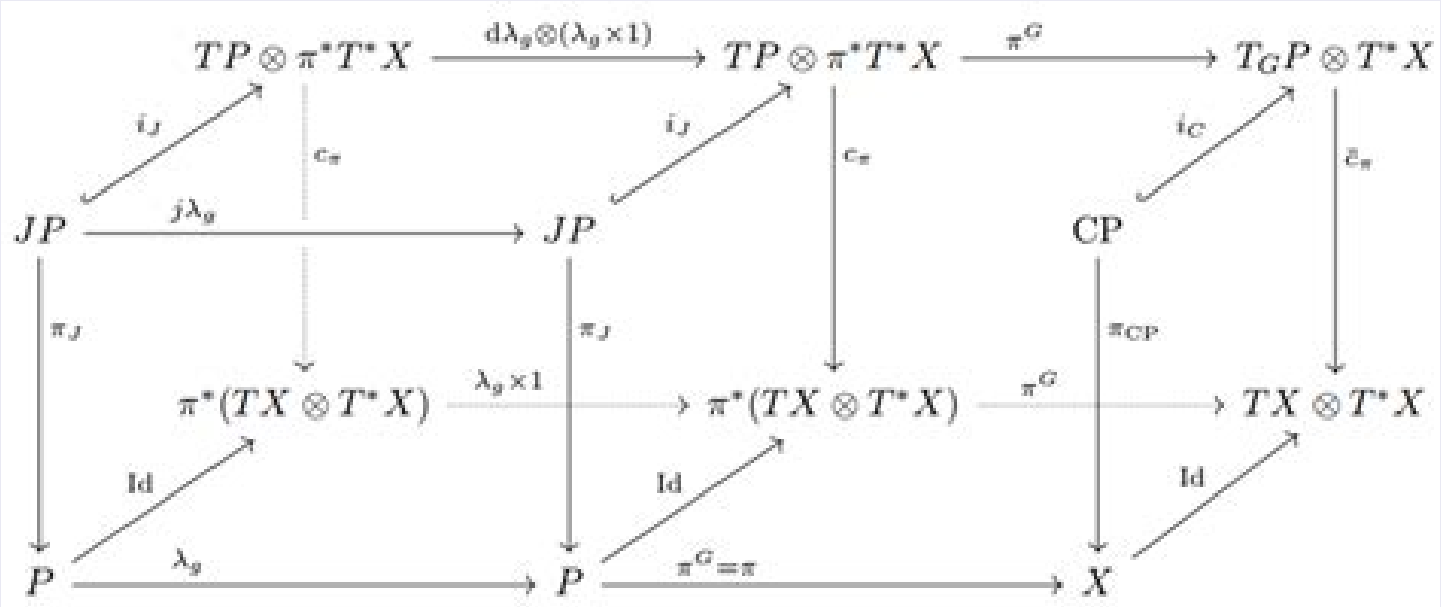
Things can get real nasty real quick



Some nice constructions

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Breaking all Limits

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- Product
- Initial/Terminal object
- Kernel (Also Nöther's Isomorphism Theorems)
- Unions? Intersections? Set Theory??? RELATIONS????? **Elements???????????**

You might need to learn basic geometry, mate

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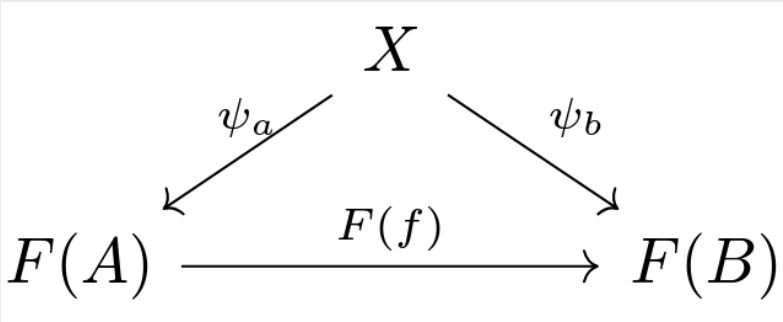
That's a funny looking cone



Tu Cone hai be?

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There are some cones that all other cones factor through. They are the *ultimate* proxy for that structure.

Excuse me, factoring through...

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Breaking all Limits

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- Limits are essentially cones that all other cones factor through.
- It is possible to have multiple of these, but they're all necessarily isomorphic!
- They are the algebraic realisation of *universal* properties.

Paulie want da Product (my name is Jeff)

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Breaking all Limits

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- Products
- Fibre Products
- Equalizers
- Direct limits
- Terminal and Initial objects
- Coproducts

A Coconut is just a nut

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Formal duals are a big deal in CT cos pretty much every construction has a categorical dual.

I can't believe I coauthored this definition

The categorical dual of a construction in $\mathcal{C}$ is the construction in $\mathcal{C}^{\mathrm{op}}$

Truly breaking all limits (down)

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Breaking all Limits

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- Does a category always have an initial object? What about the category of fields?
- How can we prove whether some limits exist in a category?

Existence theorem for limits

For a diagram $F : J \to C$, all limits of F exist in C if all equalizers and products exist in C. Equivalently,

$$\varprojlim F \cong \mathrm{eq}\left(\prod_{i \in J} F(i) \rightrightarrows \prod_{i \to j \in J} F(j)\right)$$

Wha?

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- Parametrization of functions as a map into a product $\mathbb{R} \times \mathbb{R}$.
- Limit of chain of c-rings gives the ring of p-adics. The limit is interesting because the p-adic ring is the inverse limit, which makes it *universal*
- Limits of diagrams on $\mathcal{DMG} \cong \mathrm{Psh}\left(\bullet \rightrightarrows \bullet\right)$