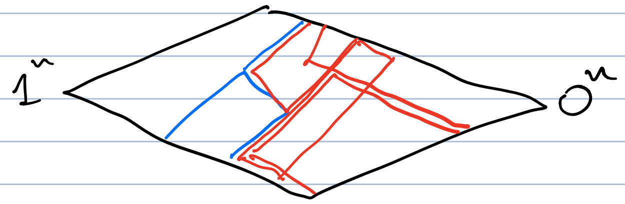
Unambiguous DNFs from Hex (joint work with Göös, Ben-David, Kothari)

A K-DNF is said to be unambiguous if

$F = C_1 \vee C_2 \vee \cdots \vee C_m$

$\forall x \in \{0,1\}^n$, there exist ≤ 1 C_i which is satisfied



FACT: Every u.a K-DNF can be written as a K^2-CNF.

Conj: This above fact is tight. $(\alpha = 2)$

TODAY: $\geq K^{1.5}$

[Balodis] $\geq K^{2-o(1)}$

$\Omega(\log^\alpha n)$

$\Omega(n^{\log^2 n})$

BOOLEAN

$\alpha = 1.12$

$= 1.22$

$f : \{0,1\}^n \to \{0,1\}$

$\boxed{P1}$ Does there exist a fn f, s.t. $C_0(f) \geq UC_1(f)^\alpha$?

$C(f,x) = \min_{\substack{\rho \in x \\ |\rho|}} f_{|\rho} = f(x) \quad \in \mathbb{N}$

$C_0[f] = \max_{x \in f^{-1}(0)} C(f,x)$

$C_0(f) =$ width of f as a CNF.

$UC_1(f) :=$ width of f " " u.a DNF.

Gap-OR $\quad x \in \{0,1\}^n$

$$f(x) = \begin{cases} 1 & \text{if } \|x\| \geq n/2 \\ 0 & \text{if } x = 0^n \\ * & \text{o/w} \end{cases}$$

$x = \underbrace{111\cdots1}_{n/2+1}\ \underbrace{000\cdots0}_{n/2-1}$

$C(f,x) = n/2$

$\boxed{1 < \alpha \leq 2}$

IDEA: Total fn are hard
Partial fn instead

$\boxed{P2}$ Does there exist a partial fn $f : \{0,1\}^n \to \{0,1,*\}$
$\exists x$, $\min\{C_{\bar{0}}(f,x), C_{\bar{1}}(f,x)\} \geq C(f)^{\alpha}$?

~~EXP~~ $C_*(f) \geq C(f)^{\alpha}$? $C(f) = \max\{C_0(f), C_1(f)\}$

f = Gap-OR

$x = \underbrace{111\cdots1}_{n/4}\ \underbrace{00\cdots0}_{3n/4}$

$f(x) = *$ $f : \to \{1,*\}$

$C_{\bar{0}}(f,x) = 1$

$C_{\bar{1}}(f,x) = n/2+1$

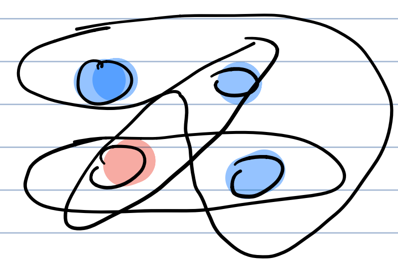
$C(f,x) \leq C_{\bar{0}}(f,x) + C_{\bar{1}}(f,x)$ $(\forall x \in f^{-1}(*))$

$\min(C_{\bar{0}}, C_{\bar{1}}) = 1$

$C_*(f) \geq C(f,x) = n/2+2$

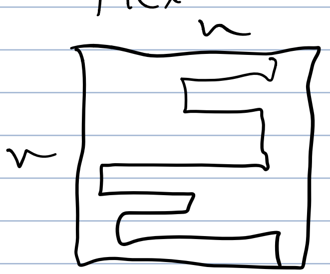
$\boxed{P3}$ Does there exist a interesting hypergraph $\mathcal{H}$ and a 2 coloring $c : V \to \{0,1\}$ s.t.

Every c-monochromatic hitting set has size $\geq r(\mathcal{H})^{\alpha}$?



Sol^n P2

Hex



Alice places 1
Bob places 0

Alice wins if she has a 1-path $\updownarrow$
Bob wins " he " " 0-path $\leftrightarrow$

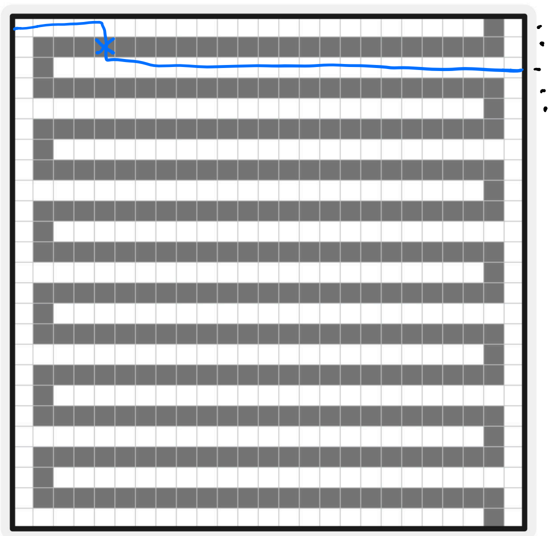
$\mathrm{Hex}: \{0,1\}^{n^2} \to \{0,1,*\}$

$$\mathrm{Hex}(n) = \begin{cases} 1 & \text{if Alice wins \& her path} \le 2n \\ 0 & \text{if Bob ''} \quad\quad\quad\quad\quad \le 2n \\ * & \text{o/w} \end{cases}$$

$C_1(\mathrm{Hex}) = ?$ $\quad 2n$

$C_0(\mathrm{Hex}) = ?$ $\quad 2n$

For $\boxed{P2}$, $\exists\, z$, $\min(C_0(z), C_1(z)) \ge n^\alpha$?

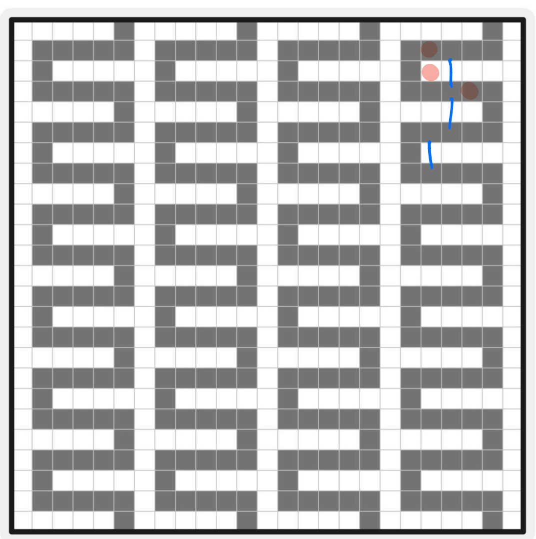


$\frac{n^2}{2} - n \le C_0 \le n^2/2$

$\Omega(n) \le C_1 \le 5n$

$\min(C_0, C_1) \in O(n)$

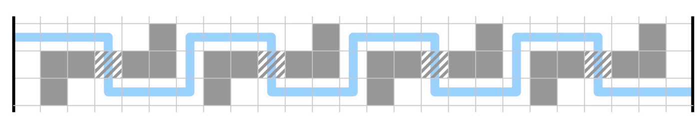
1-Spiral



$\Omega(n\sqrt{n}) \le C_0 \le n\sqrt{n}$

$\Omega(n\sqrt{n}) \le C_1 \le O(n\sqrt{n})$

$\min(\ ) \ge n^{1.5}$



$\sqrt{n}$ copies, $\sqrt{n}$-width 1-Spirals