

# Approximate Computing

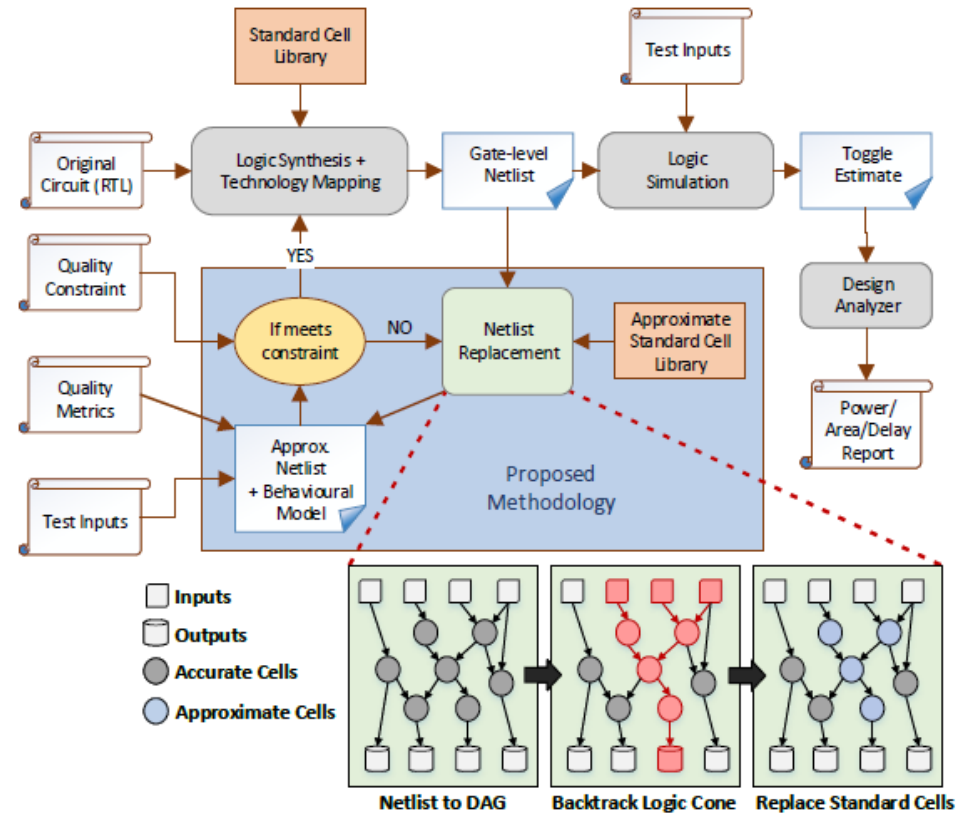
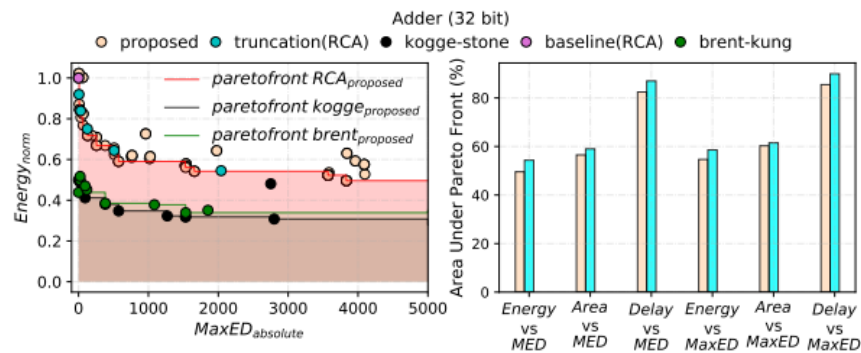
Salman Ahmed

# Motivation and Ad-hoc Approaches

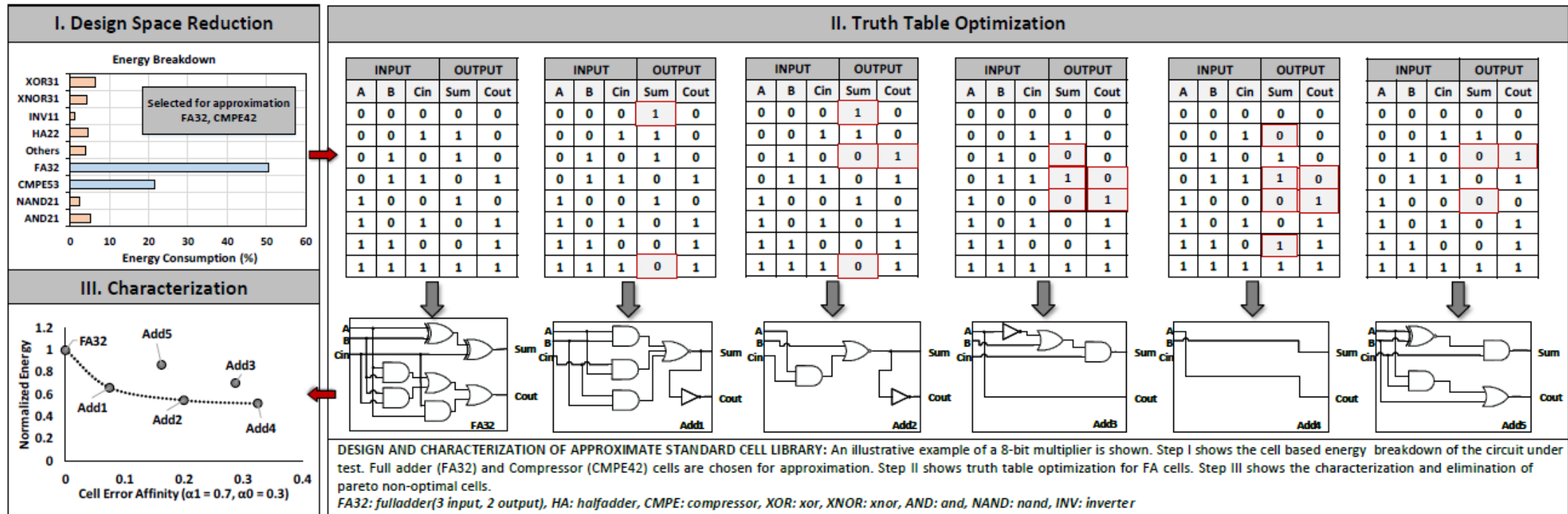
- Declining Moore's law
- Dark Silicon
- Noisy input
- Limited data precision
- Physical defects
- Additional load or hard real-time constraints
- Perceptual limitations of humans
- Trade off quality with efficiency
- Precision Scaling
- Loop Perforation
- Load Value Approximation
- Memoization
- Skipping Tasks and Memory Accesses
- Using Multiple Inexact Program Versions
- Using Inexact or Faulty Hardware
- Using Voltage Scaling
- Approximating DRAM Memories

# ALS: Using Approximate Std. Cell Library

- Design Space Reduction
- Truth table Optimization
- Characterization



# ALS: Using Approximate Std. Cell Library



# ALS: Using AST Transformations

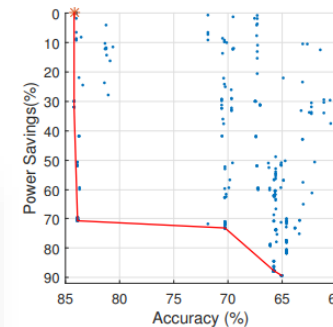
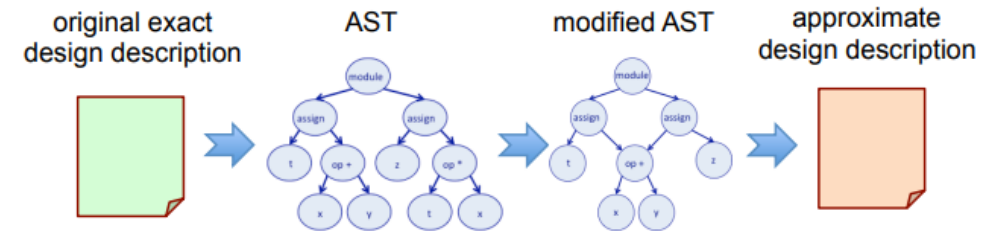
- Data Type Simplifications
- Operation Transformations
- Arithmetic Expression Transformations
- Variable-to-Constant Substitution Transformations
- Loop Transformations

```

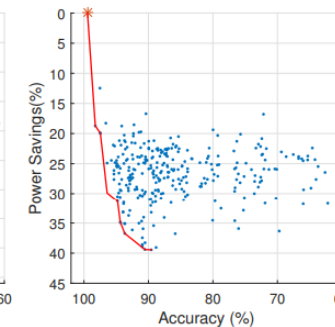
always @(*)
begin
sum_temp[0]=result[0];
for (i=1; i<nSVs; i=i+1)
begin
sum_temp[i] = sum_temp[i-1] + result[i];
end
end
    
```

```

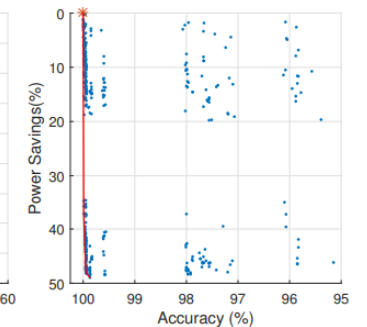
always @(*)
begin
sum_temp[0]=(sum_temp[2] | result[3]);
sum_temp[1]=(sum_temp[0] | result[1]);
sum_temp[2]=(sum_temp[1] | result[2]);
sum_temp[3]={{result[0][15:3], {3'b000}}};
sum_temp[4]=(sum_temp[3] + result[4]);
sum_temp[5]=(sum_temp[4] + result[5]);
end
    
```



(a) Perceptron



(b) FIR Filter

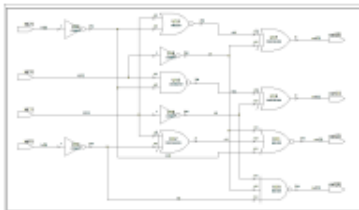


(c) FFT

# ALS: Using Boolean Matrix Factorization

original circuit

	z 1	z 2	z 3	z 4
0000	0	0	0	1
0001	1	0	0	1
0010	1	0	1	1
0011	1	0	1	1
0100	0	0	0	0
0101	1	0	0	0
0110	1	0	1	1
0111	1	0	1	1
1000	1	0	1	0
1001	1	0	1	0
1010	1	0	0	0
1011	1	0	0	0
1100	1	0	0	1
1101	1	1	0	1
1110	1	1	1	0
1111	1	0	1	0



area = 22.3  $\mu\text{m}^2$

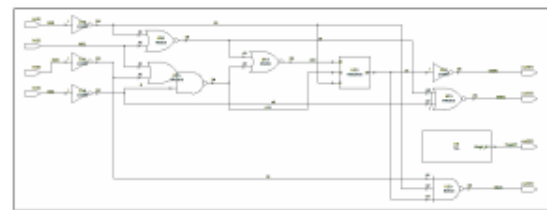
approximate circuit (f = 3)

compressor			decompressor			
t 1	t 2	t 3	z 1	z 2	z 3	z 4
0	0	0	1	0	1	0
0	1	0	1	0	0	1
1	1	0	1	0	0	0
1	1	0				
1	1	0				
0	0	0				
0	0	1				
1	1	0				
1	1	0				
1	0	0				
1	0	0				
0	0	1				
0	0	1				
0	1	0				
0	1	0				
1	0	0				
1	0	0				

=

z 1	z 2	z 3	z 4
0	0	0	0
1	0	0	1
1	0	1	1
1	0	1	1
0	0	0	0
1	0	0	0
1	0	1	1
1	0	1	1
1	0	1	0
1	0	1	0
1	0	0	0
1	0	0	0
1	0	0	1
1	0	0	1
1	0	1	0
1	0	1	0

Hamming distance 3



area = 19.1  $\mu\text{m}^2$

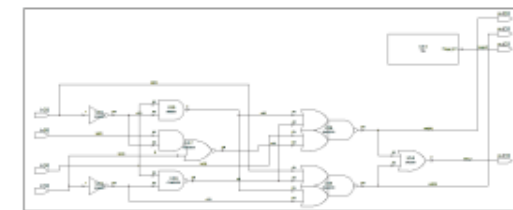
approximate circuit (f = 2)

compressor		decompressor			
t 1	t 2	z 1	z 2	z 3	z 4
0	0	1	0	1	0
0	1	1	0	0	1
1	1				
1	1				
0	0				
0	0				
1	1				
1	1				
1	0				
1	0				
0	0				
0	0				
0	1				
0	1				
1	0				
1	0				

=

z 1	z 2	z 3	z 4
0	0	0	0
1	0	0	1
1	0	1	1
1	0	1	1
0	0	0	0
0	0	0	0
1	0	1	1
1	0	1	1
1	0	1	0
1	0	1	0
0	0	0	0
0	0	0	0
1	0	0	1
1	0	0	1
1	0	1	0
1	0	1	0

Hamming distance 6



area = 16.2  $\mu\text{m}^2$

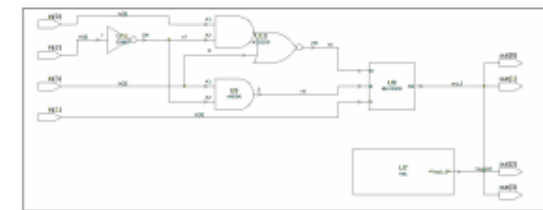
approximate circuit (f = 1)

compressor	decompressor			
t 1	z 1	z 2	z 3	z 4
0	1	0	1	1
1				
1				
1				
0				
0				
1				
1				
1				
1				
0				
0				
1				
1				
1				

=

z 1	z 2	z 3	z 4
0	0	0	0
1	0	1	1
1	0	1	1
1	0	1	1
0	0	0	0
0	0	0	0
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1
0	0	0	0
0	0	0	0
1	0	1	1
1	0	1	1
1	0	1	1

Hamming distance 13



area = 9.4  $\mu\text{m}^2$

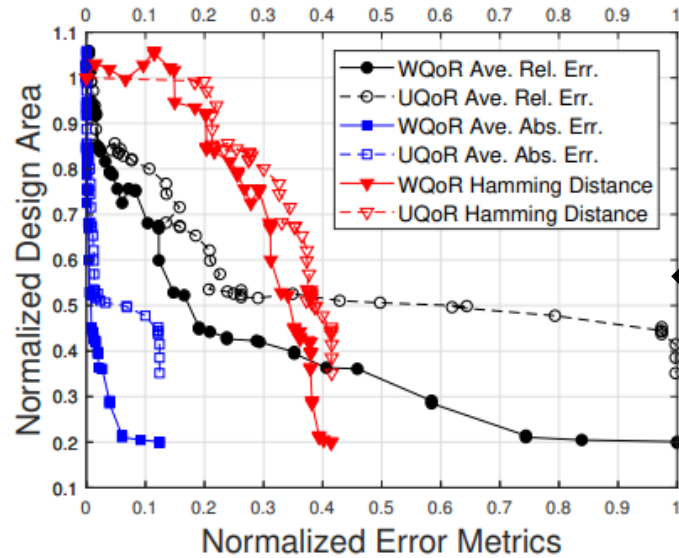
# ALS: Using Boolean Matrix Factorization

## Algorithm 1: BLASYS: Boolean Level Approximate Circuit Synthesis

```

Input : Accurate Circuit  $ACir$ , Error Threshold
Output: Approximate Circuit  $Cir$ 
1 subcircuits=Decompose input circuit using  $k \times m$  decomposition
2 // Factorization profiling Phase
3 for each subcircuit  $s_i$  with  $m_i \leq m$  outputs do
4    $M$ =Construct truth table of  $s_i$ 
5   // profile for every possible factorization degree
6   for  $f=1$  to  $m_i-1$  do
7      $[B, C] = BMF(M, f)$ 
8      $T_{s_i, f}$ =Construct truth table of  $BC$ 
9   end
10 end
11 // Circuit Space Exploration Phase
12  $Cir=ACir$ ;
13 Let  $f_i = m_i$  for all subcircuits  $s_i$ 
14 while  $QoR(Cir) < threshold$  do
15   for each subcircuit  $s_i$  with  $f_i > 1$  do
16      $Cir' = Cir(s_i \rightarrow T_{s_i, f_i-1})$ 
17      $\Delta err_i = QoR(Cir') - QoR(Cir)$ 
18   end
19    $b = \arg \min_i(\Delta err_i)$ 
20    $Cir = Cir(s_b \rightarrow T_{s_b, f_b-1})$ 
21    $f_b = f_b - 1$ 
22 end
23  $Cir$ =Synthesize Best new Design
24 return  $Cir$ 

```



Constraint:  
Minimization of  
 $\|(M - S_1 \cdot S_2)w\|_2$   
for target QoR

Design	Area Savings (%)	Power Savings (%)	Delay Reduction (%)
Adder32	44.78	63.79	12.07
Mult8	28.77	26.87	12.32
BUT	7.87	11.25	2.23
MAC	47.55	55.58	64.41
SAD	32.80	41.47	69.14
FIR	19.52	22.26	12.18

# ALS: Using Pre-trained Error Network

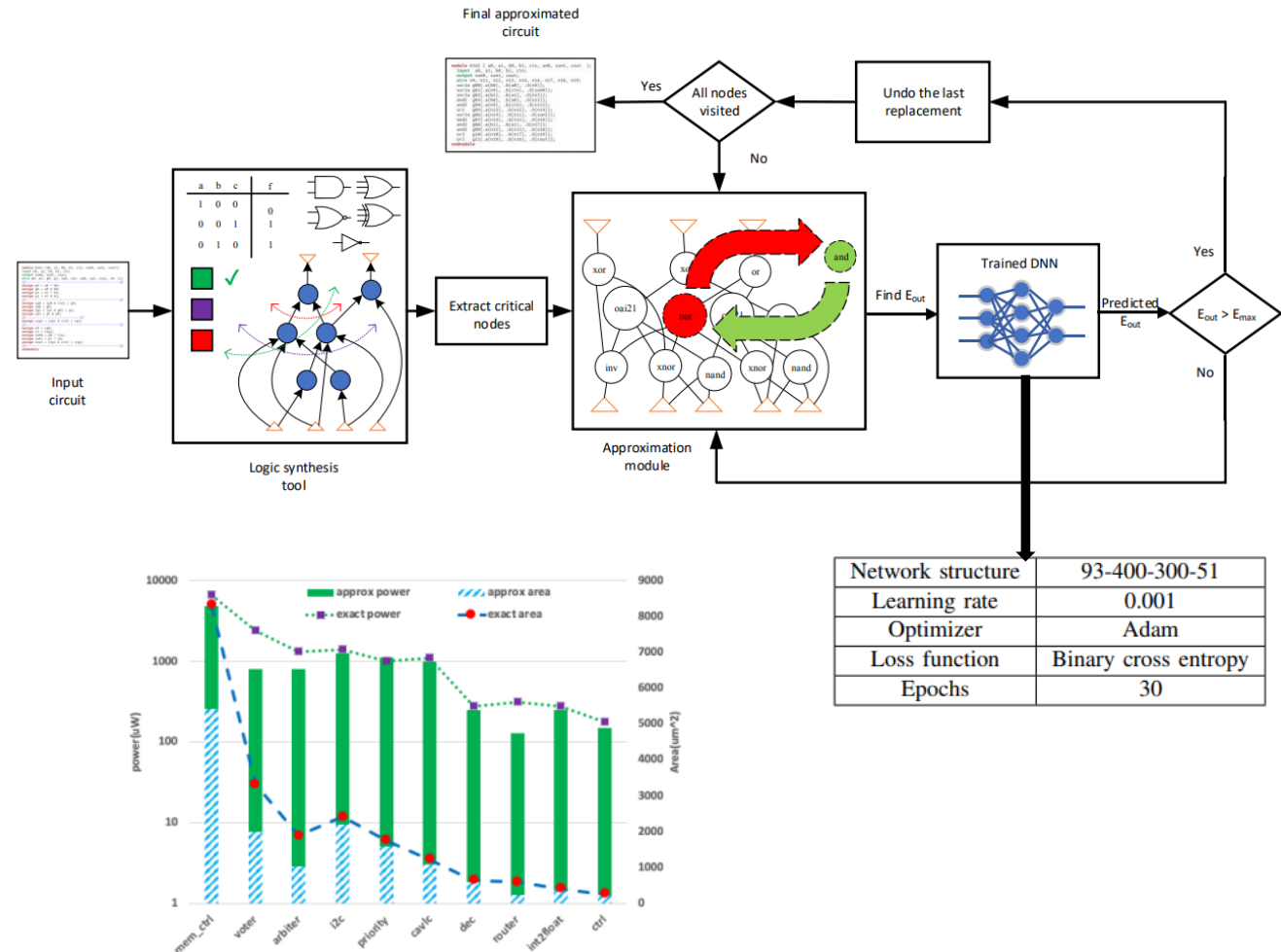
## Algorithm 2: Deep-PowerX Inference

**Input:** Input circuit  $G = (V, E)$ , Technology library, Max. error constraint ( $E_{max}$ )  
**Output:** Approx. circuit  $G_X = (V_X, E_X)$

```

// Initialization:
1  $G_X = \text{Copy } G$ .
2  $V' = \text{Extract top 20\% of active nodes of } V_X$ .
3  $V'' = \text{Extract top 20\% immediate fanouts of nodes in } V'$ .
4 set  $E_{out}$  to 0.0, and copy technology library to  $Lib$ .
// Minimizing power:
5  $Lib1$  ( $Lib2$ ) = Sort  $Lib$  in ascending order by gates' out (in) cap.
6  $opt\_mode = \text{power}$ ;
7  $V''' = V' \cup V''$ 
8 for each node  $n$  in  $V'''$  do
9   if  $opt\_mode$  is power then
10    if  $n \in V'$  then
11       $Lib = Lib1$ ;
12    else if  $n \in V''$  then
13       $Lib = Lib2$ ;
14  for each gate  $g$  in  $Lib$  do
15    Replace  $n$  with  $g$ ; Utilize DNN to infer  $E_{pred}$ 
16    if replacement is accepted then
17      Update  $E_{out}$  with  $E_{pred}$ .
18      Break;
19    else
20      Undo the last gate replacement.
21  if  $E_{out} > E_{max}$  then
22    Undo the last gate replacement.
23    Break;
// Minimizing area:
24 if  $E_{out} < E_{max}$  then
25    $Lib = \text{Sort } Lib$  in ascending order by gates' area.
26    $opt\_mode = \text{area}$ ;
27   // Copy nodes of approx. circuit in power
28   // minimization phase:
29    $V''' = V_X$ ; Go to line 8.
28 return  $G_X$ 

```



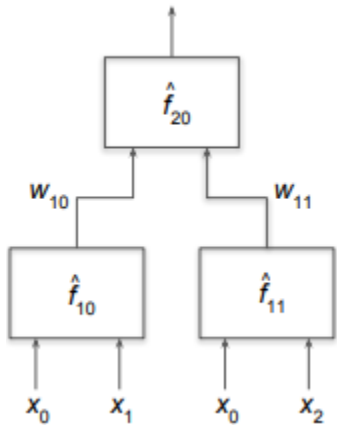


# ALS: Using LUT Memorization

$$\hat{f} : \mathbb{B}^k \rightarrow \mathbb{B} \quad b \in \mathbb{B}$$

$$\hat{f}(p) = \begin{cases} 1 & \text{if } c_{p1} > c_{p0}, \\ 0 & \text{if } c_{p1} < c_{p0}, \\ b & \text{if } c_{p1} = c_{p0} \end{cases}$$

$x$ $x_0x_1x_2$	$y$	$p$ $x_0x_1x_2$	$y^0$	$y^1$	$p$	$\hat{f}$
000	0	000	1	2	000	1
000	1	001	0	1	001	1
000	1	010	0	0	010	0*
001	1	011	0	0	011	1*
100	0	100	1	0	100	0
110	0	101	0	0	101	1*
110	1	110	1	1	110	1*
110	1	111	0	0	111	0*



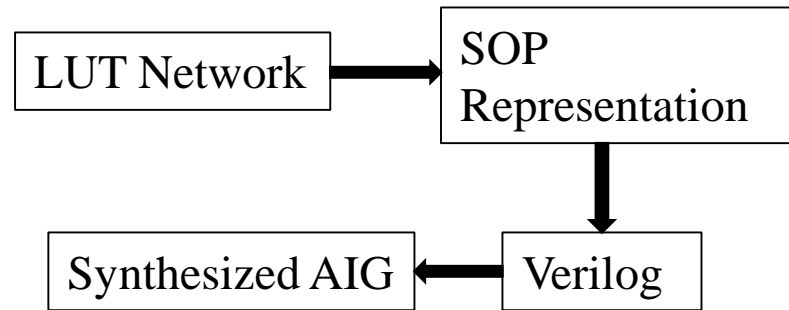
$p$ $x_0x_1$	$y^0$	$y^1$	$\hat{f}_{10}$
00	1	3	1
01	0	0	1*
10	1	0	0
11	1	1	1*

$p$ $x_0x_2$	$y^0$	$y^1$	$\hat{f}_{11}$
00	1	2	1
01	0	1	1
10	2	1	0
11	0	0	1*

$x$ $x_0x_1x_2$	$w_{10}w_{11}$	$y$
000	11	0
000	11	1
000	11	1
001	11	1
100	00	0
110	10	0
110	10	1

$p$ $w_{10}w_{11}$	$y^0$	$y^1$	$\hat{f}_{20}$
00	1	0	0
01	0	0	1*
10	1	1	0*
11	1	3	1

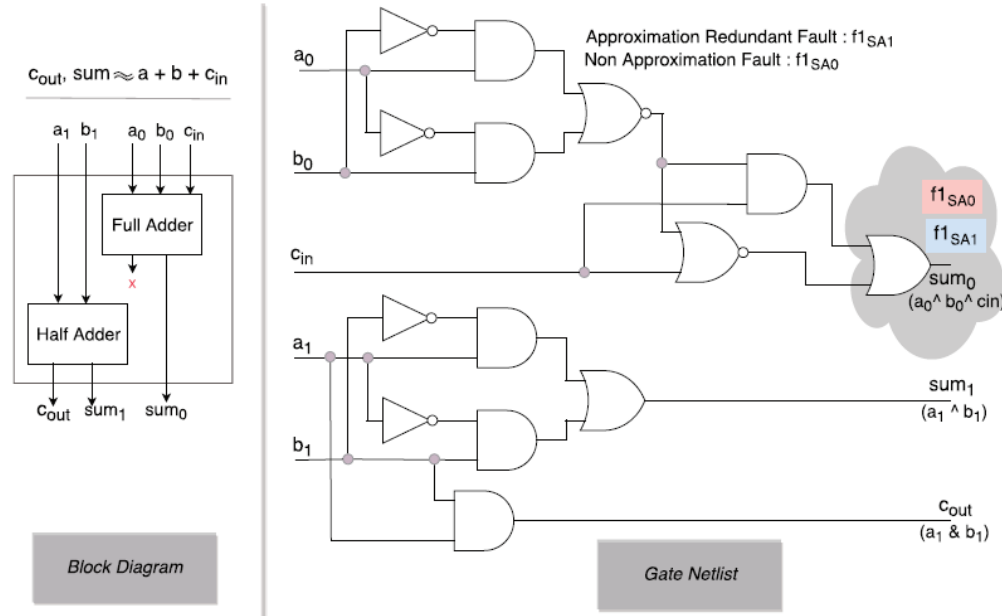
# ALS: Using LUT Memorization



LAYER	LUT COUNT	TRAINING ACCURACY			
		MEAN	STD	MIN	MAX
0	784	0.5072	0.0340	0.4042	0.6572
1	1024	0.6055	0.0403	0.5120	0.7299
2	1024	0.7431	0.0191	0.6721	0.7877
3	1024	0.8297	0.0068	0.8038	0.8526
4	1024	0.8655	0.0033	0.8562	0.8751
5	1024	0.8808	0.0015	0.8759	0.8853
6	1	0.8898	0.0000	0.8898	0.8898

	PLANE	AUTO	BIRD	CAT	DEER	DOG	FROG	HORSE	SHIP	TRUCK
PLANE		0.77	0.77	0.80	0.82	0.81	0.84	0.81	0.71	0.79
AUTO	0.96		0.79	0.77	0.79	0.80	0.80	0.78	0.76	0.67
BIRD	0.95	0.98		0.68	0.63	0.69	0.66	0.72	0.83	0.81
CAT	0.96	0.98	0.96		0.70	0.61	0.68	0.71	0.81	0.76
DEER	0.96	0.98	0.95	0.96		0.73	0.69	0.71	0.83	0.81
DOG	0.98	0.99	0.97	0.96	0.97		0.72	0.70	0.82	0.79
FROG	0.97	0.98	0.95	0.95	0.97	0.96		0.75	0.85	0.80
HORSE	0.98	0.99	0.96	0.97	0.96	0.98	0.97		0.81	0.75
SHIP	0.93	0.96	0.98	0.98	0.98	0.98	0.99	0.98		0.77
TRUCK	0.97	0.95	0.98	0.97	0.97	0.98	0.98	0.97	0.98	

# Test of Approximate Circuits



ISCAS circuits <sup>4</sup>	#gates	$f_{orig}$	$f_{final}^{bf}$	$f_{\Delta}^{bf}$ (%)	sec
c499*	577	1320	1153	12.65%	73s
c880*	527	1074	305	71.60%	31s
c1355*	575	1330	1196	10.08%	79s
c1908*	427	974	949	2.57%	30s
c2670*	931	1950	428	78.05%	396s
c3540*	1192	2657	839	68.42%	418s
c5315*	2063	4224	1648	60.98%	6224s
c6288*	2836	7048	3071	56.42%	4881s

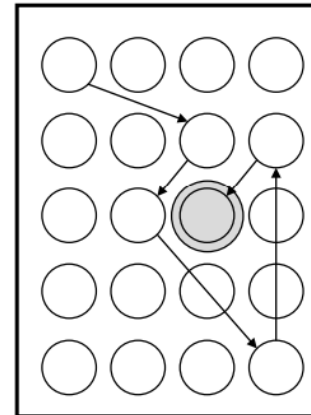
Correct <sup>†</sup>		Appx <sup>‡</sup>		Appx:SA0*		Appx:SA1 <sup>±</sup>	
In	Out <sup>†</sup>	Out <sup>‡</sup>	$e^{\ddagger}$	Out*	$e^*$	Out <sup>±</sup>	$e^{\pm}$
00000	000	000	0	000	0	001	1
00001	001	001	0	000	1	001	0
00010	010	010	0	010	0	011	1
00011	011	011	0	010	1	011	0
00100	001	001	0	000	1	001	0
00101	010	000	2	000	2	001	1
00110	011	011	0	010	1	011	0
00111	100	010	2	010	2	011	1
01000	010	010	0	010	0	011	1
01001	011	011	0	010	1	011	0
01010	100	100	0	100	0	101	1
01011	101	101	0	100	1	101	0
01100	011	011	0	010	1	011	0
01101	100	010	2	010	2	011	1
01110	101	101	0	100	1	101	0
01111	110	100	2	100	2	101	1
10000	001	001	0	000	1	001	0
10001	010	000	2	000	2	001	1
10010	011	011	0	010	1	011	0
10011	100	010	2	010	2	011	1
10100	010	000	2	000	2	001	1
10101	011	001	2	000	3	001	2
10110	100	010	2	010	2	011	1
10111	101	011	2	010	3	011	2
11000	011	011	0	010	1	011	0
11001	100	010	2	010	2	011	1
11010	101	101	0	100	1	101	0
11011	110	100	2	100	2	101	1
11100	100	010	2	010	2	011	1
11101	101	011	2	010	2	011	2
11110	110	100	2	100	2	101	1
11111	111	101	2	100	3	101	2

†, ‡ Golden non-approx, approx (carry cut) 2-bit adder responses|  
 \*, ± Approx adder with SA0, SA1 at  $sum_0$  ( $f_{1SA0}$ ,  $f_{1SA1}$ )  
 In:  $C_{in} a_1 a_0 b_1 b_0$ , Out:  $C_{out} sum_1 sum_0$   
 $e$ : error in each case, worst-case errors  $wc^{\ddagger}=2, wc^*=3, wc^{\pm}=2$

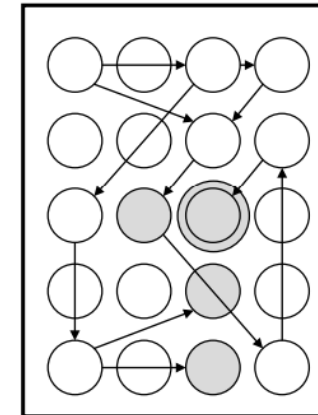
# Insecurity of Approximate Circuits

Attack model	Key features	
No. 1: tamper interconnect	Assumption	(1) AC functional IP is a blackbox; (2) Interconnect between AC and non-AC IPs are accessible;
	Attack method	(1) Swap MSB and LSB bits; (2) Force LSB to stuck-at-0/1;
No. 2: tamper AC function	Assumption	(1) AC functional IP is a whitebox; (2) Non-AC IPs are protected blackbox;
	Attack method	(1) Use hardware Trojan to trigger malicious approximate function; (2) Use external control to alter ambient environment.

Unlocking sequence in a deterministic circuit



Unlocking sequence in a non-deterministic circuit



F. Regazzoni, C. Alippi and I. Polian, "Security: The Dark Side of Approximate Computing?," *2018 IEEE/ACM International Conference on Computer-Aided Design (ICCAD)*, 2018, pp. 1-6.  
 Pruthvy Yellu, Mezanur Rahman Monjur, Timothy Kammerer, Dongpeng Xu, Qiaoyan Yu, "Security Threats and Countermeasures for Approximate Arithmetic Computing", *Design Automation Conference (ASP-DAC) 2020 25th Asia and South Pacific*, pp. 259-264, 2020.