How fast do quantum walks mix?

Shantanav Chakraborty[1], Kyle Luh[2] and Jérémie Roland[1]

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1 QuIC, Université libre de Bruxelles
2 CMSA, Harvard University

IIIT Theory Group Saturday Seminars, IIIT Hyderabad
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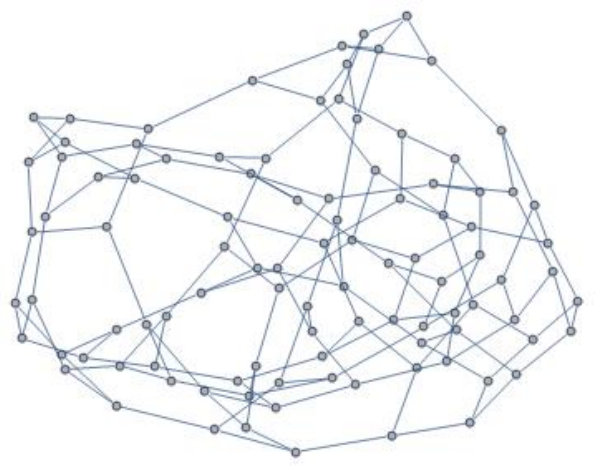
Mixing of Classical random walk on a graph



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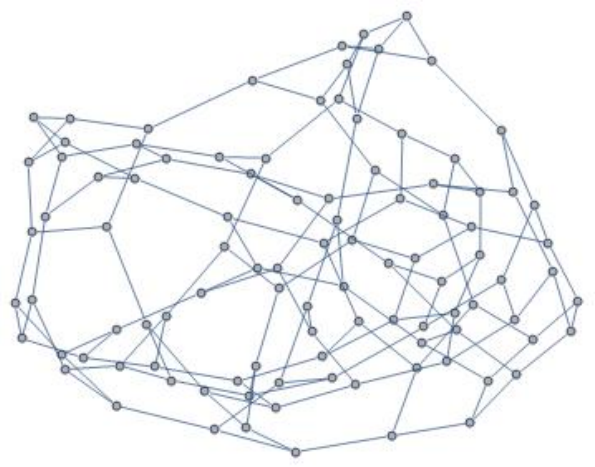

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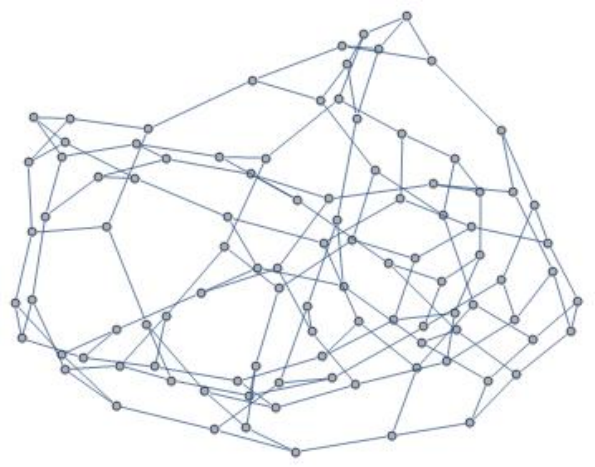

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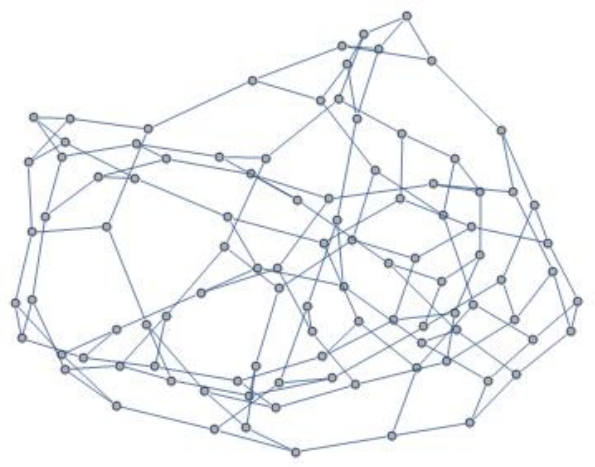

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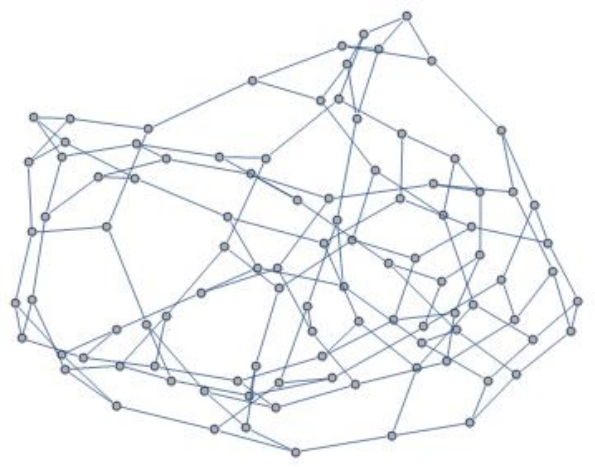

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- π is independent of v_0.

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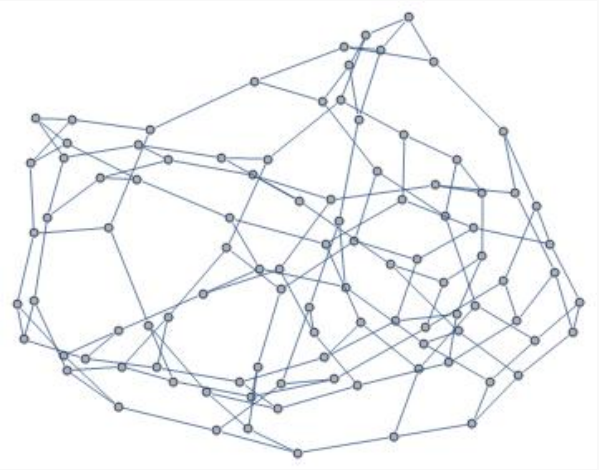

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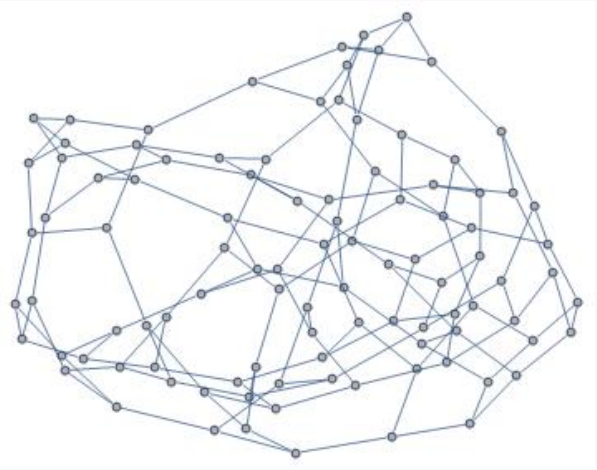

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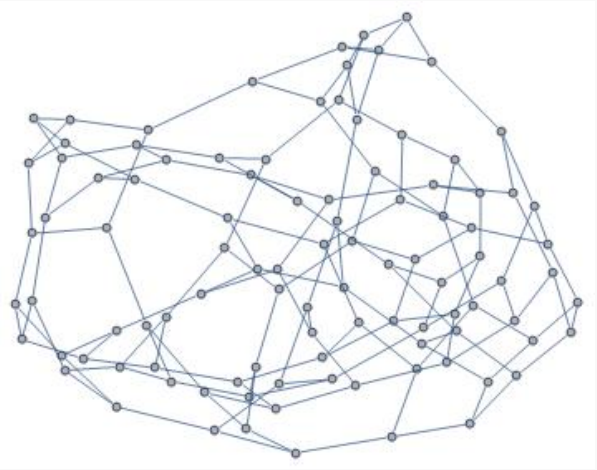

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If P has eigenvalues: $\lambda_n = 1 > \lambda_{n-1} \geq \cdots \lambda_1 \geq -1$,

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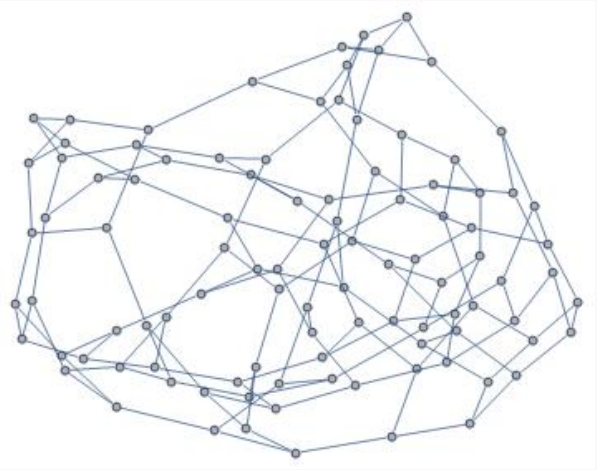

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- Depends only on the spectral gap
- Independent of the initial distribution of the walker.

All the Quantum Mechanics you'll need for this talk:

(i) Any quantum state can be represented by a (unit) column vector in a Hilbert space, denoted as $|.\rangle$.

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In fact, c_j denotes the "*probability amplitude*": probability of $|\psi_0\rangle$ to be in the state $|j\rangle$ is given by the absolute square of the inner-product (or overlap) between the states $|j\rangle$ and $|\psi_0\rangle$:

$$p_j = |\langle j|\psi_0\rangle|^2 = |c_j|^2. \qquad [|c_j| \leq 1]$$

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$$\left|\left\langle j|M_j|\psi_t\right\rangle\right|^2 = \left|\left\langle j|\psi_t\right\rangle\right|^2.$$

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  - This will be our focus!
  - We will consider continuous-time quantum walks, but results are valid for its discrete-time counterpart.

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- We shall consider the normalized adjacency matrix as the quantum walk Hamiltonian: $\bar{A}_G = \gamma A_G$, where $\gamma = 1/\|A_G\|$.
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Upper bound on the quantum mixing time:

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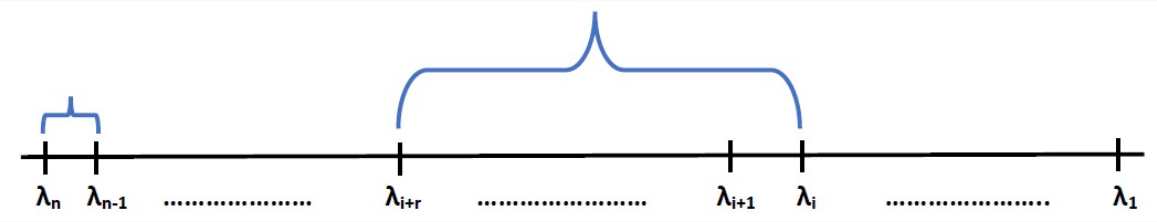

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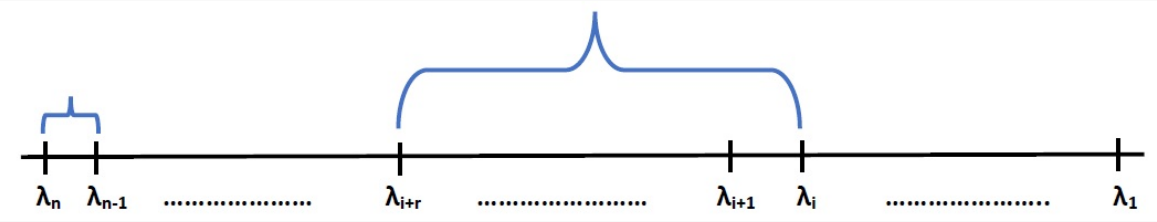

We need to bound

$$\Sigma = \sum_{i=1}^{n-1}\frac{1}{|\lambda_{i+1}-\lambda_i|} + \sum_{i=1}^{n-2}\frac{1}{|\lambda_{i+2}-\lambda_i|} + \ldots..$$

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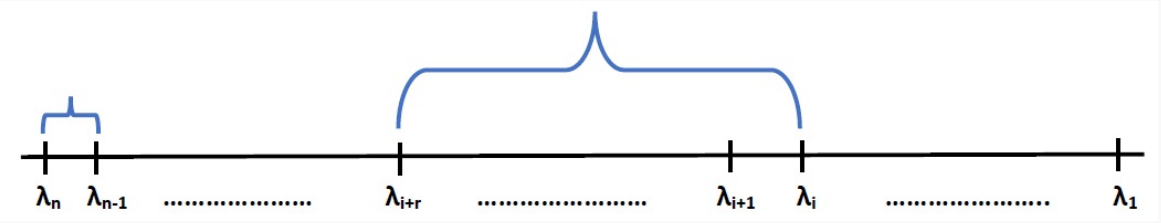

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So $T^G_{\text{mix}} = \widetilde{O}\,(n/\Delta_{\text{min}})$ is an upper bound for all graphs.

For cycles, the mixing time is in $\widetilde{O}(n)$. [AAKV 2001].

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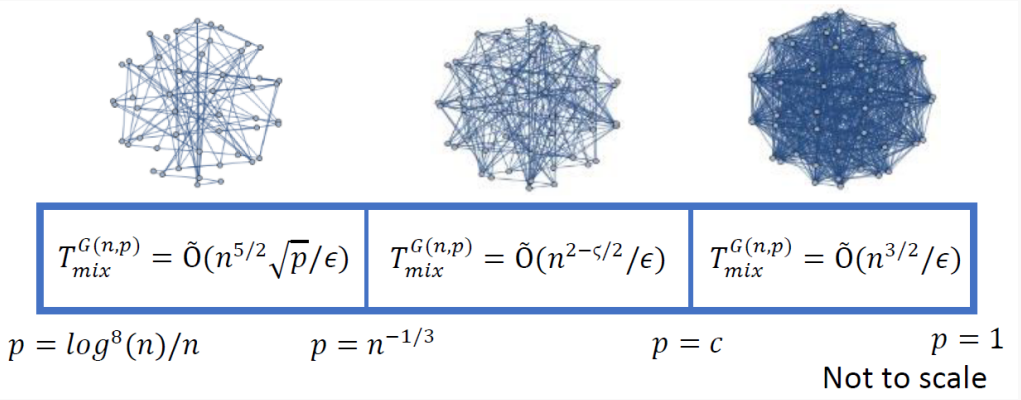

$p \geq n^{-1+\zeta}$, $\zeta \geq 2/3$.

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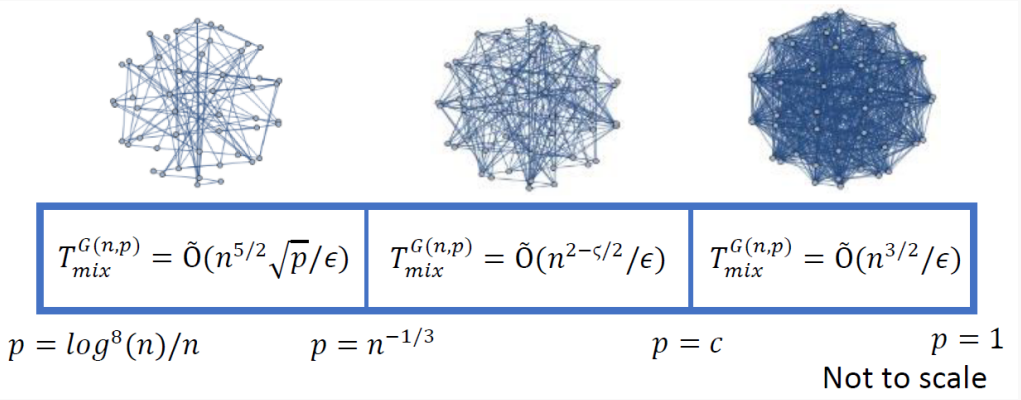
Can we obtain a better upper bound for the mixing time for general classes of graphs?



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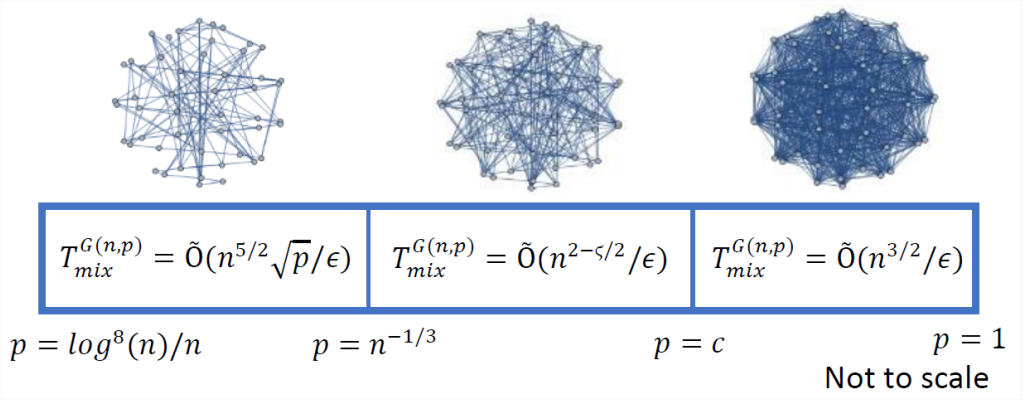

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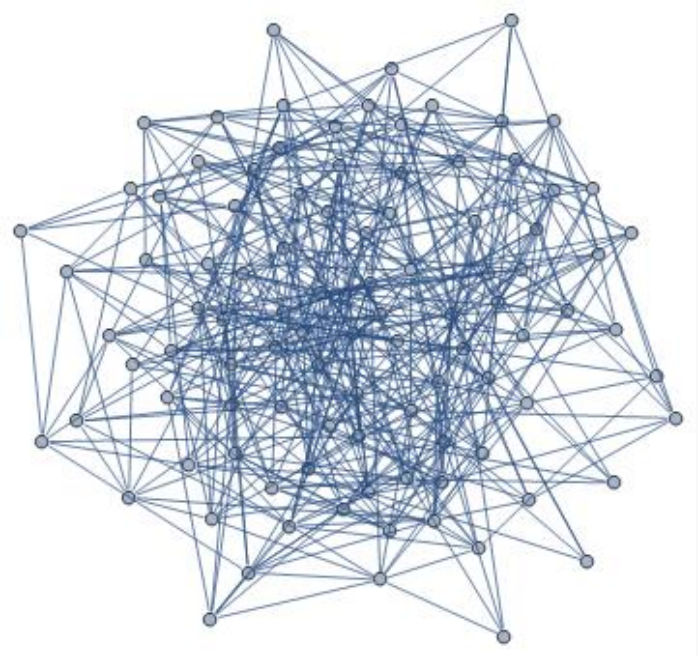
- More generally, we obtain a better upper bound on the quantum mixing time for sparse and dense Erdös-Renyi random graphs. Bound holds for *almost all* graphs!

Erdös-Renyi random graph (Erdös-Renyi 1959)



A graph where each edge exists with probability p independently of the other edges. Denoted as $G(n, p)$.

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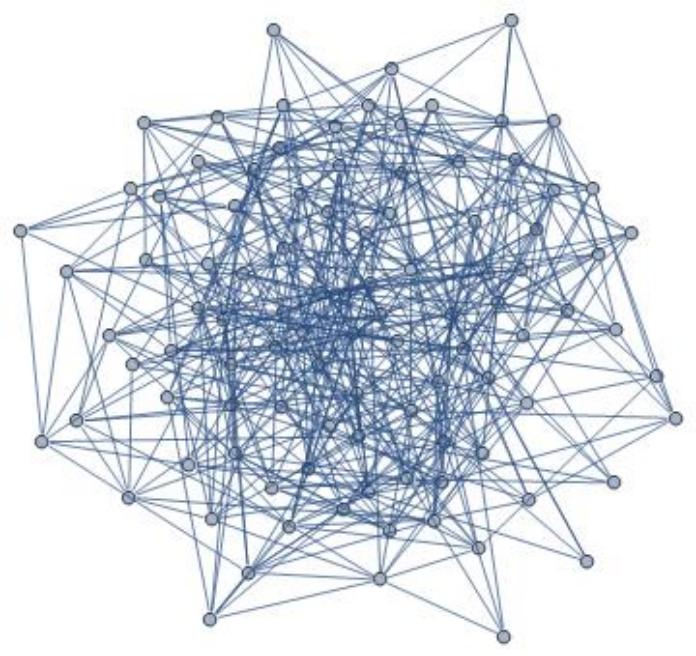

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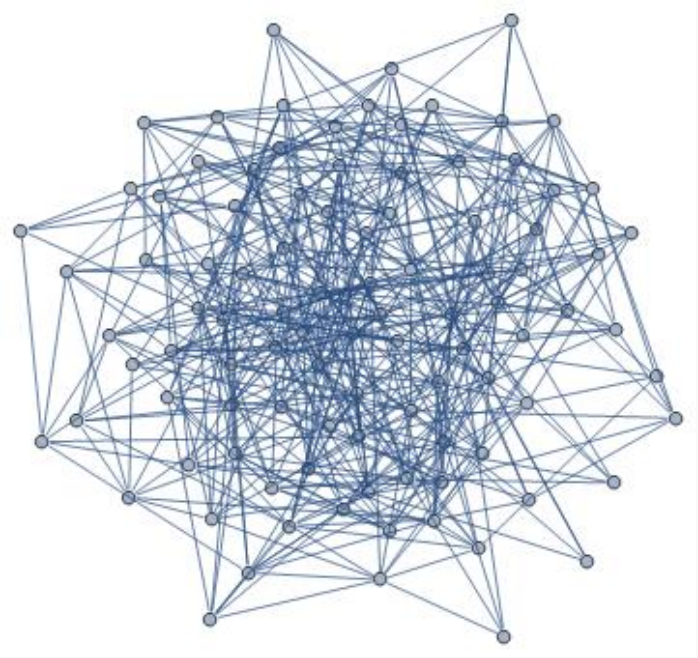

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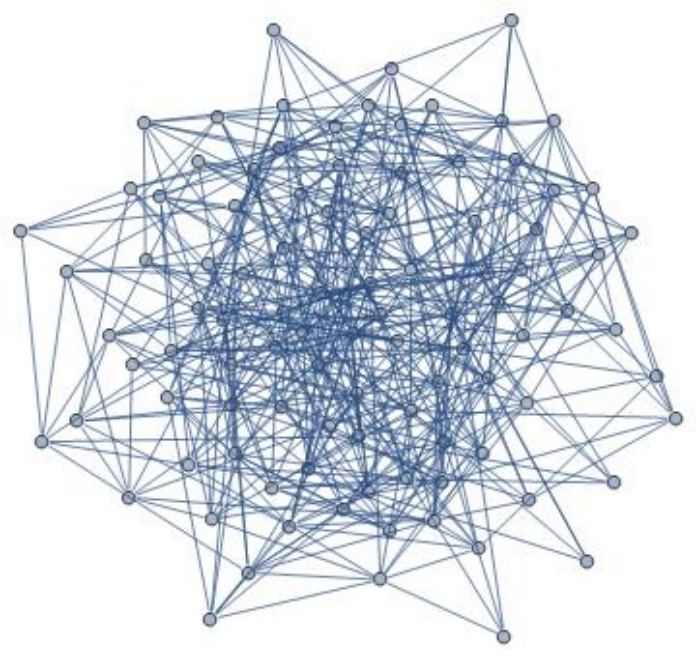

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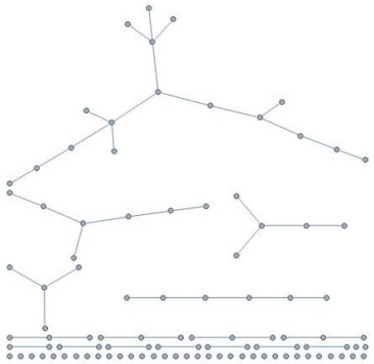
- ***Almost all*** graphs satisfy property P, if $G(n, 1/2)$ satisfies P with probability $1 - o(1)$.
- Fraction of graphs satisfying P goes to 1 as $n \to \infty$.

Evolution of Erdös-Renyi random graphs

p=1/n

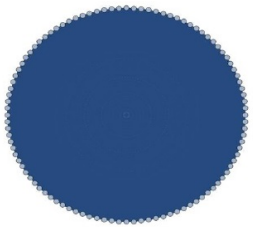


Isolated trees, paths and nodes

p=ln(n)/n

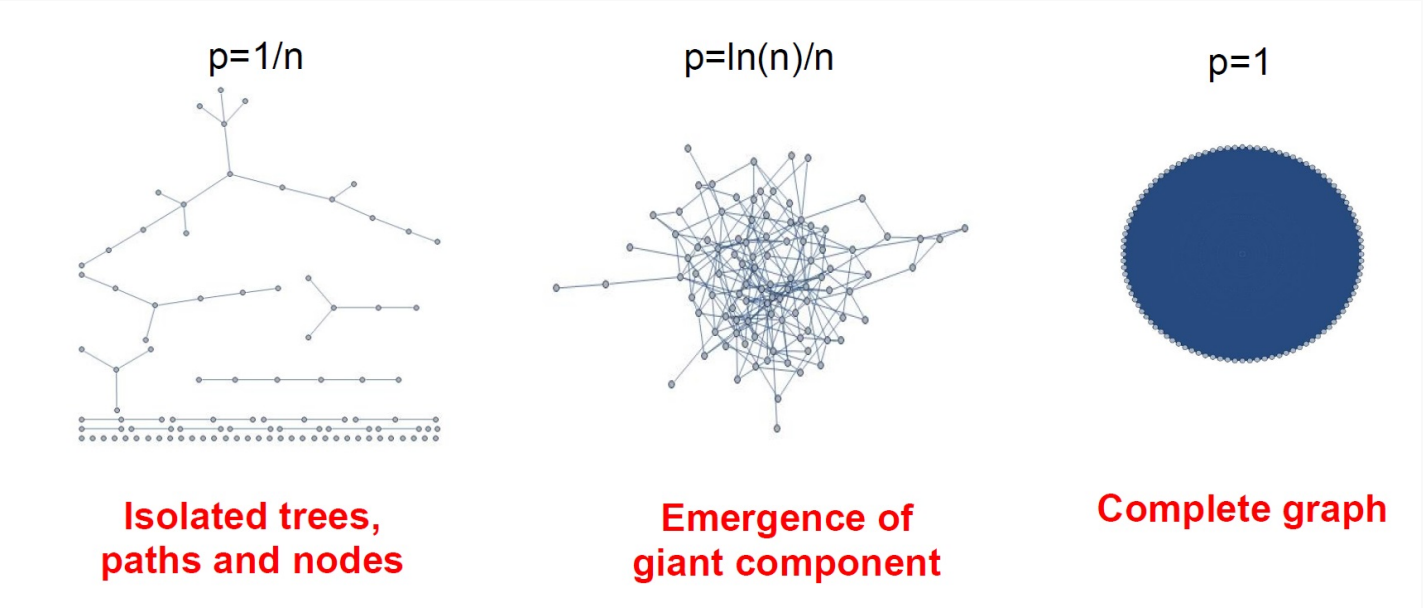
Emergence of giant component

p=1



Complete graph

Evolution of Erdös-Renyi random graphs



$A_{G(n,p)}$ is a symmetric random matrix: Each (non-diagonal) entry is 1 with probability p and 0 with probability $1-p$.

Upper bound on the quantum mixing time of $G(n,p)$:

We prove that for $p \geq n^{-1/3}$,

$$T_{\text{mix}}^{G(n,p)} = \widetilde{\mathcal{O}}\left(n^{3/2-\log(p)/\log(n)}\sqrt{p}\right),$$

almost surely.

Spectral properties of $A_{G(n,p)}$

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- Spectral gap $\Delta = \Omega(1) \implies$ Classical mixing time is in $\widetilde{O}(1)$.
- For the quantum mixing time, all gaps are crucial.

Wigner's semicircle distribution

- As long as $np \to \infty$, the bulk of the spectrum of $A_{G(n,p)}$ follows the *Wigner's semicircle distribution:*

$$\rho_{sc}(x) = \begin{cases} \dfrac{\sqrt{4np(1-p) - x^2}}{2\pi np(1-p)} & \text{if } |x| < 2\sqrt{np(1-p)} \\ 0 & \text{otherwise} \end{cases}.$$

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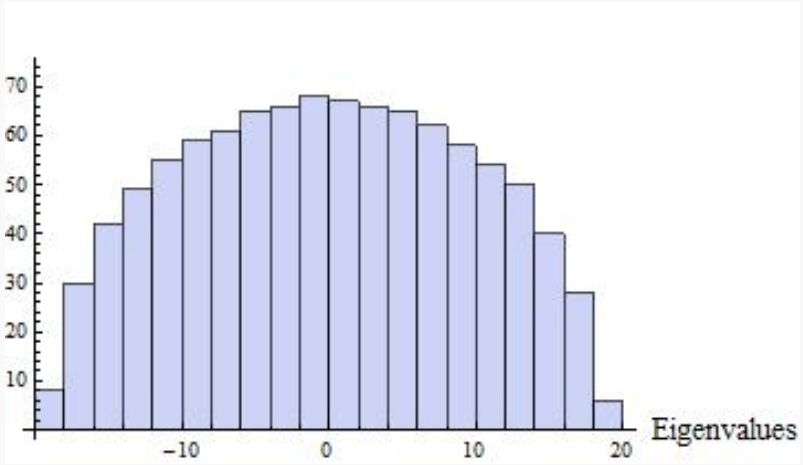
- Radius of the semicircle: $R = 2\sqrt{np(1-p)}$. For $\bar{A}_{G(n,p)}$: $R = 2\sqrt{\frac{1-p}{np}}$.
- Divide $[-R, R]$ into small bins and count the eigenvalues of $A_{G(n,p)}$ in each bin:

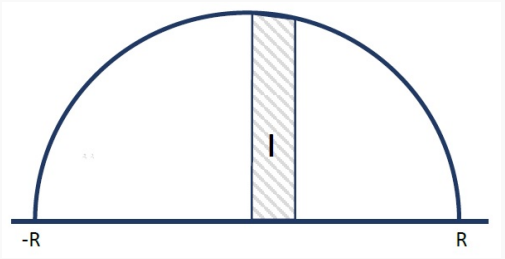


This distribution will converge to a semicircle.

Wigner's semicircle distribution

For any interval $\mathcal{I} \in \mathbb{R}$ if $\mathcal{N}_{\mathcal{I}}$ is the number of eigenvalues of $A_{G(n,p)}$ in $\mathcal{I}$, then

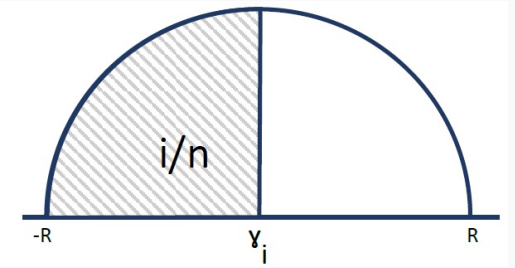
$$\frac{\mathcal{N}_{\mathcal{I}}}{n} = \int_{\mathcal{I}} \rho_{sc}(x)dx + o(1).$$



Classical eigenvalue location:

For any index $1 \leq i \leq n-1$, the *classical location* of each eigenvalue λ_i, denoted by γ_i, is given by

$$\int_{-R}^{\gamma_i} \rho_{sc}(x)dx = \frac{i}{n}.$$



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- *Macroscopically*, each eigenvalue should be close to their classical locations.
- Works only when $|\mathcal{I}| \gg 1$ and not when say $|\mathcal{I}| \sim 1/n$.
- Useless when we need bounds on consecutive gaps, i.e. $\lambda_{i+1} - \lambda_i$.

What will we need?

Average eigenvalue gap:

$$\bar{\Delta}_{G(n,p)} = \Theta\left(\frac{1}{n^{3/2}\sqrt{p}}\right)$$

Distance between classical eigenvalue locations: For $i \leq n/2$, $r \leq n - 2i$ and some universal constant $c > 0$

$$\gamma_{i+r} - \gamma_i \geq c\frac{r}{n^{7/6} i^{1/3}\sqrt{p}}.$$

Eigenvalue rigidity criterion

Eigenvalues of $\bar{A}_{G(n,p)}$ are concentrated around their classical locations. We present a simplified version (adapted for our analysis):

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Eigenvalue rigidity criterion [EYY2011, EKYY2013]:

For $1 \leq i \leq n-1$, any $\varepsilon \geq 0$ and $p \geq n^{-1/3}$, the eigenvalues of $\bar{A}_{G(n,p)}$ satisfy the inequalities

$$|\lambda_i - \gamma_i| \leq \frac{n^{\varepsilon}(n^{-2/3}\alpha_i^{-1/3} + n^{-\phi})}{(pn)^{1/2}}$$

with probability $1 - o(1)$, where

$$\phi := \frac{\log pn}{\log n} \quad \text{and} \quad \alpha_i := \max\{i, n-i\}.$$

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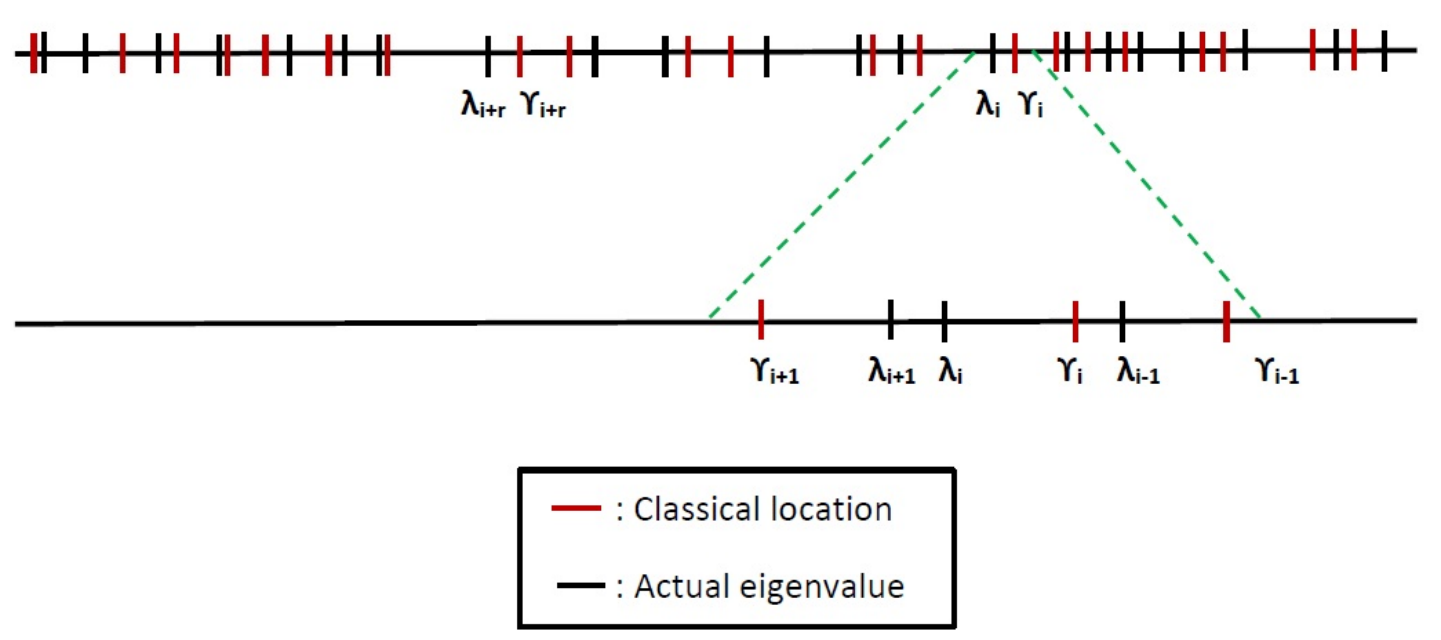

Microscopic statistics of eigenvalues

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- What about bounds on consecutive eigenvalue gaps, i.e. $\delta_i = \lambda_{i+1} - \lambda_i$?
- Notoriously difficult problems in random matrix theory.
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- Not only this, Nguyen, Tao and Vu [NTV2015] showed that each δ_i is separated.
- They obtain tail bounds on δ_i: How likely is each δ_i to be δ times less than the average?

Tail-bounds on eigenvalue gaps of $\bar{A}_{G(n,p)}$ [NTV2015, LL2019]:

For $p \geq \log^6(n)/n$ and $1 \leq i \leq n-1$

$$\sup_{1 \leq i \leq n-1} \mathbb{P}\left(\delta_i \leq \delta \frac{1}{n^{3/2}\sqrt{p}}\right) \leq C\delta \log n,$$

with probability $1 - o(1)$, where $\delta \geq n^{-C}$

Microscopic statistics of eigenvalues $\bar{A}_{G(n,p)}$

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Applying union bound gives a lower bound on $\Delta_{\min}$.

Minimum eigenvalue gap $\bar{A}_{G(n,p)}$:

$$\Delta_{\min} \geq \frac{1}{n^{5/2+o(1)}\sqrt{p}},$$

with probability $1 - o(1)$.

Upper bounding Σ_1

Recall:

$$\Sigma = \sum_{i=1}^{n-1} \sum_{r=1}^{n-i} \frac{1}{|\lambda_{i+r} - \lambda_i|},$$

and $1/\Delta_{\min} \leq \Sigma \leq \widetilde{\mathcal{O}}\left(n/\Delta_{\min}\right)$.

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First we prove an upper bound on Σ_1 using the tail bounds on δ_i:

Upper bound on Σ_1:

$$\Sigma_1 = \sum_{i=1}^{n-1} \frac{1}{|\lambda_{i+1} - \lambda_i|} \leq n^{5/2+o(1)}\sqrt{p},$$

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Key idea: Many δ_i's are close to the average. Count them!

- Σ_1 is close to $1/\Delta_{\min}$.

Upper bound on Σ

To now obtain an upper bound on Σ, we combine two things:

(i) Distance between classical eigenvalue locations:

$$\gamma_{i+r} - \gamma_i \geq c \frac{r}{n^{7/6} i^{1/3} \sqrt{p}}.$$

(ii) Eigenvalue rigidity - Eigenvalues are close to the classical locations

$$|\lambda_i - \gamma_i| \leq \frac{n^{\varepsilon}(n^{-2/3}\alpha_i^{-1/3} + n^{-\phi})}{(pn)^{1/2}}$$

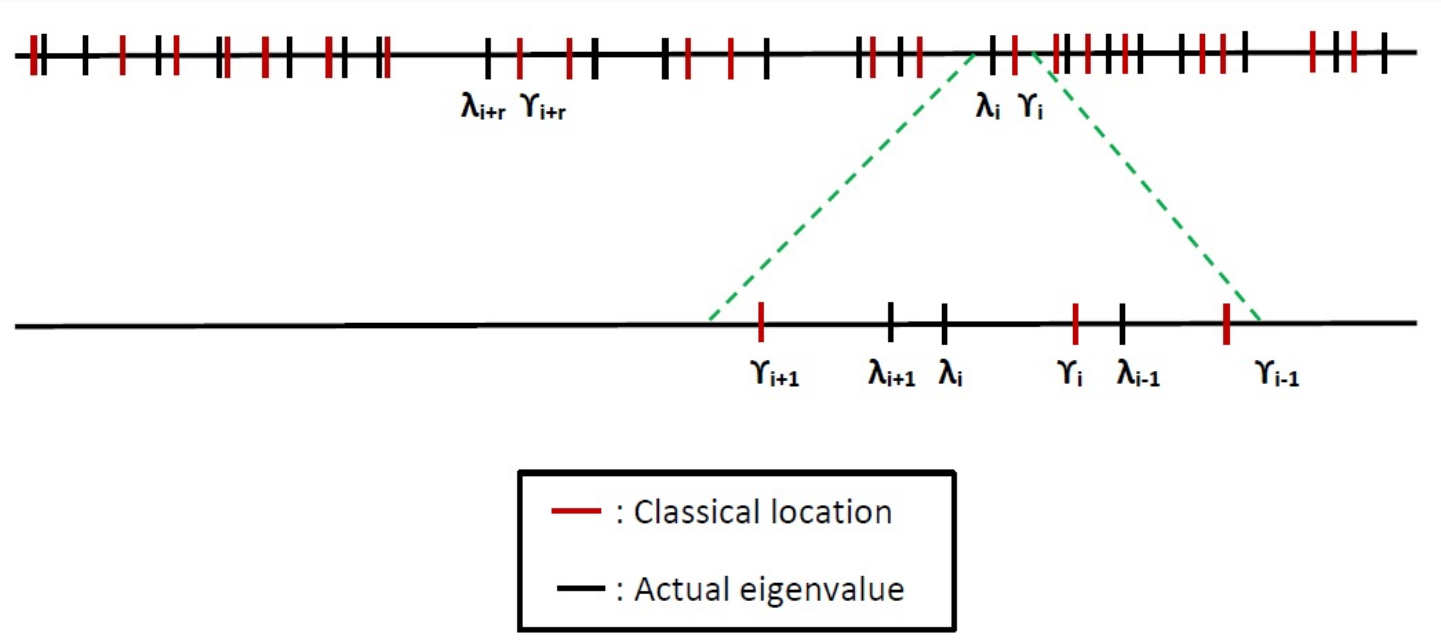
Idea:

For small values of r use tailbounds of δ_i while for larger values of r make use of eigenvalue rigidity!

Upper bound on Σ

Idea:

- For small values of r use tailbounds of δ_i while for larger values of r make use of eigenvalue rigidity!
- Exploit eigenvalue rigidity for large enough r such that $(\gamma_{i+r} - \gamma_i)$ is larger than the error due to $|\lambda_{i+r} - \gamma_{i+r}| + |\lambda_i - \gamma_i|$.
- Critical value: $r_\star(i) \leq n^{\varepsilon - \log p / \log n}$.



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Proof idea: Split Σ into two parts.

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$$\leq r_\star(i) \cdot \Sigma_1 + (np)^{1/2} \sum_{r=1}^{n-1} \sum_{i=1}^{n-r} \frac{C n^{2/3} i^{1/3}}{r}.$$

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Mixing time of random graphs

Recall:

Limiting distribution:

$$P_f(T \to \infty) = \lim_{T\to\infty} P_f(T) = \sum_{i=1}^{n} |\langle f|v_i\rangle \langle v_i|\psi_0\rangle|^2.$$

- Our results are independent of $|\psi_0\rangle$.
- All eigenstates of $\bar{A}_{G(n,p)}$ are delocalized, i.e. $\||v_i\rangle\|_\infty \leq n^{-1/2+o(1)}$, with probability $1 - o(1/n)$ [EKYY 2013, HKM 2018].

Limiting distribution for $G(n,p)$:

Immediately we obtain:

$$P_f(T \to \infty) \leq \widetilde{\mathcal{O}}(1/n),$$

almost surely irrespective of $|\psi_0\rangle$. (Close to uniform!)

Mixing time of random graphs

Limiting distribution:

$$P_f(T \to \infty) \leq \widetilde{\mathcal{O}}\left(1/n\right).$$

Upper bound on the quantum mixing time:

$$T_{\text{mix}}^{G} = \mathcal{O}\left(\frac{1}{\epsilon}\sum_{i=1}^{n-1}\sum_{r=1}^{n-i}\frac{|\langle v_i|\psi_0\rangle|\,.\,|\langle\psi_0|v_{i+r}\rangle|}{|\lambda_{i+r}-\lambda_i|}\right).$$

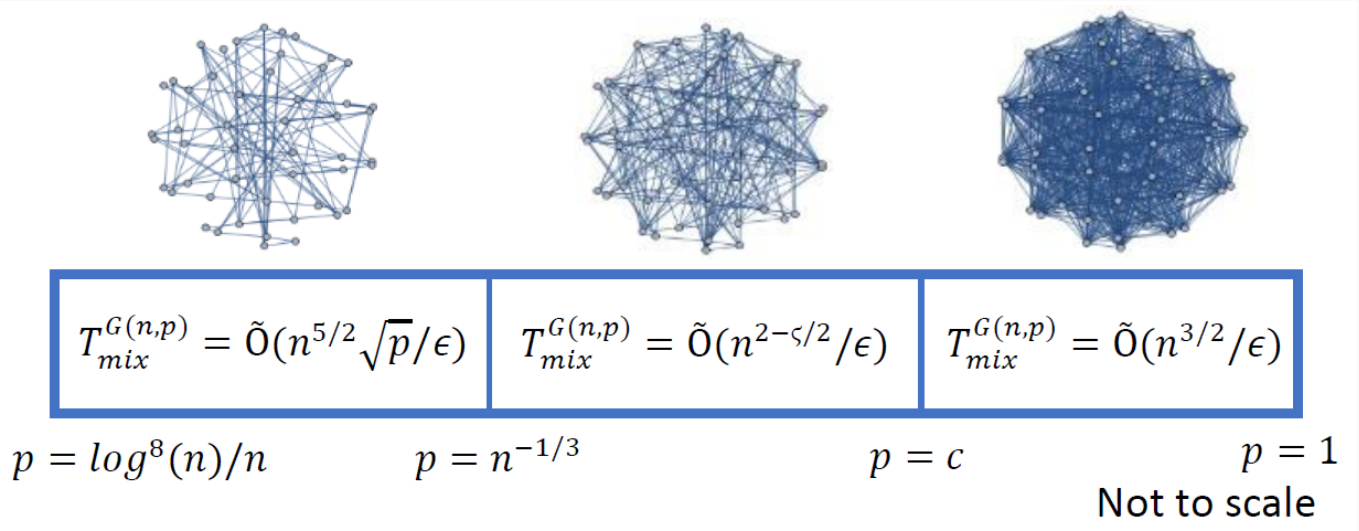
Quantum mixing time for $G(n,p)$

From the upper bound on Σ_1 and from the delocalization of the eigenstates we have

$$T_{\text{mix}}^{G(n,p)} = \widetilde{\mathcal{O}}\left(n^{3/2-\log(p)/\log(n)}\sqrt{p}/\epsilon\right),$$

almost surely for any $p \geq n^{-1/3}$.

Summary of results



$p \geq n^{-1+\zeta}$, where $\zeta \geq 2/3$.

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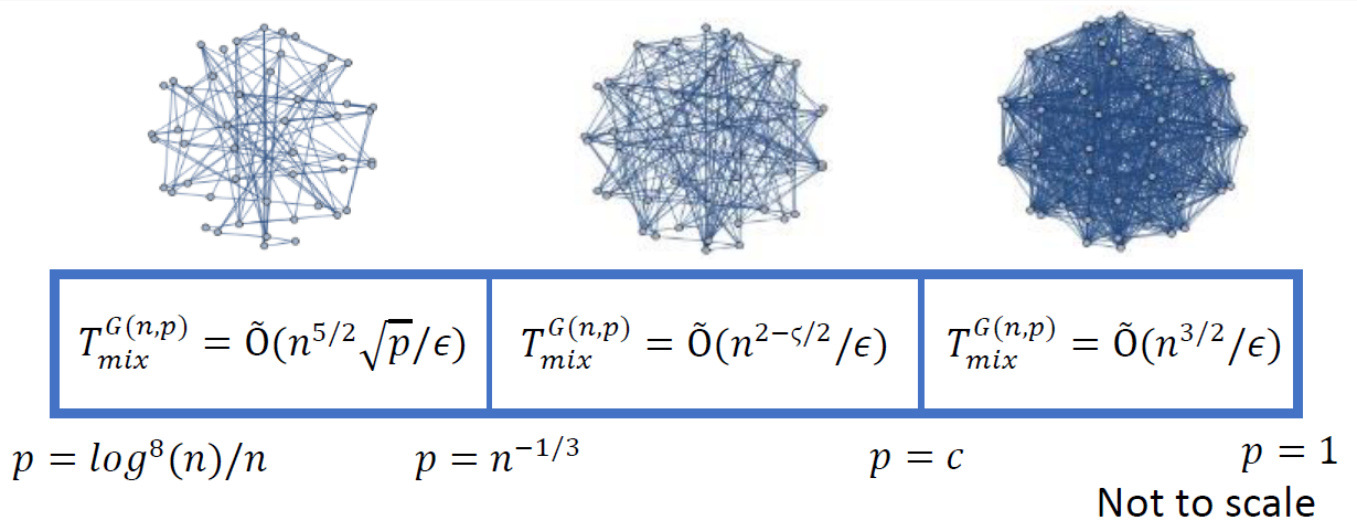

$p \geq n^{-1+\zeta}$, where $\zeta \geq 2/3$.

- Improved upper bound on the quantum mixing time for almost all graphs.
- For sparser graphs, i.e. when $p = \log^D(n)/n,\ D \geq 8$, eigenvalue rigidity breaks down.
- In that case we have a weaker upper bound of

$$T_{\text{mix}}^{G(n,p)} = \widetilde{O}(n^{5/2}\sqrt{p}/\epsilon).$$

Ongoing and Future work

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- Our techniques can be used to improve bounds on the equilibration times of isolated quantum systems defined by random Hamiltonians.
- Randomized time evolution can be harnessed to improve the performance of several quantum walk based algorithms. E.g.: Running time of the Glued trees algorithm of Childs et al. (STOC 2003), $\mathcal{O}(n^5) \to \widetilde{\mathcal{O}}(n^2)$.
  [*C and Y. Atia, arXiv:2005.04062 (2020)*].

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- Discrete-time quantum walks W using continuous-time quantum walks H: Express

$$W^K \approx \sum_{t=-\mathcal{O}(K)}^{+\mathcal{O}(K)} c_t e^{-iHt}.$$

Thank you for your attention!